MATHEMATICAL PLAY: ACROSS AGES, CONTEXT, AND CONTENT

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For the third time in four years, the Mathematical Play Working Group met to discuss the state of research into the role that play can have in mathematics education research. This meeting provided an opportunity for members within the group to share recent progress in their respective areas of focus. The group also continued theoretical discussions regarding areas of focus for new research and productive framings and perspectives for addressing mathematical play. Additionally, the working group dedicated a portion of the meeting to collaborating with the Embodied Mathematical Imagination and Cognition [EMIC] *Working Group.*

Keywords: Instructional activities and practices; Affect, emotion, beliefs & attitudes; Informal education

For the past four years, the Mathematical Play working group has convened to investigate and advance the field's understanding of mathematical play. Our collaborative work has been characterized by: 1) experiencing mathematical play together, 2) identifying the features and affordances of mathematical play for learning and doing mathematics, 3) and discussing implications for designing mathematical play learning experiences across ages and contexts. Broadly, our work has spanned both theoretical and practical worlds; for example, we seek to understand what aspects of mathematical activity might allow it to be characterized as "play" while simultaneously considering the extent to which mathematical play is possible and productive in classrooms. Our explorations have ranged in context from early childhood preschool classrooms to university mathematics courses, and have examined the use of block play, videogames, partner activities, simulations, board games, and making/tinkering activities.

Summary of 2021 Meeting

Our goal for the 2021 working group meeting was to examine mathematical play along two dimensions - activity grounded in instruction and activity grounded in play - with specific exploration of activities that might support shifts along these dimensions in two specific ways (Figure 1). On the first day, we explored how traditional mathematical instructional tasks might be altered to afford greater opportunity for play. On the second day, we investigated ways play-based activities might be adapted to better support meaningful learning toward specific conceptual outcomes. Working group sessions engaged participants in conceptualizing these two shifts through sharing and explorations of existing projects and discussions of perspectives to support such shifts. Finally, we spent the third day of the meeting collaborating with the Embodied Mathematical Imagination and Cognition [EMIC] Working Group.

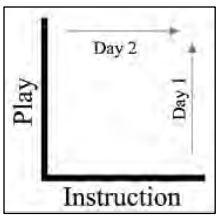


Figure 1: Organization of two shifts we will explore – increasing the playfulness of high instructional tasks (Day 1) and increasing the instructional utility of high play tasks (Day 2)

Day 1: Adapting Mathematical Tasks to Increase Opportunities for Play

On the first day of the 2021 working group, Ellis and Plaxco each presented examples from their respective projects demonstrating efforts to shift existing activities grounded in instruction toward more playful settings. Ellis presented a new (as of the conference) project in which her research team is modifying existing tasks designed to support covariational reasoning about linear and quadratic relationships for play activities. This new area is based on Ellis' extensive work supporting students' sensemaking regarding rates of change and accumulating growth (e.g., Ellis, Ely, Singleton, & Tasova, 2020). In her group's attempt to play-ify existing tasks, Ellis described scenarios in which players (with experience creating accumulation graphs from a given growing shape) would build accumulation graphs and for other players who were then tasked with reconstructing the growing shape. Working group attendees participated in a paired activity to play this game with each other.

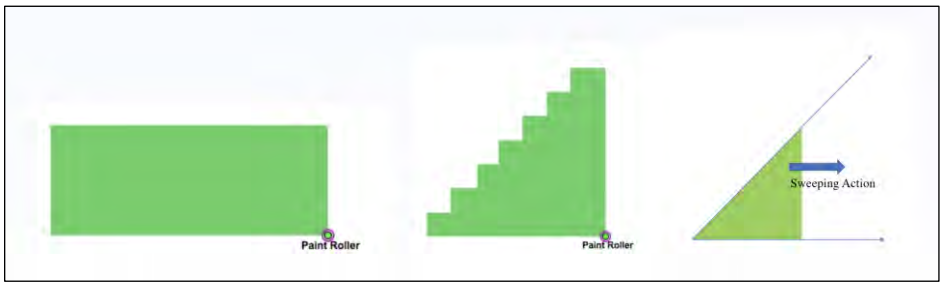
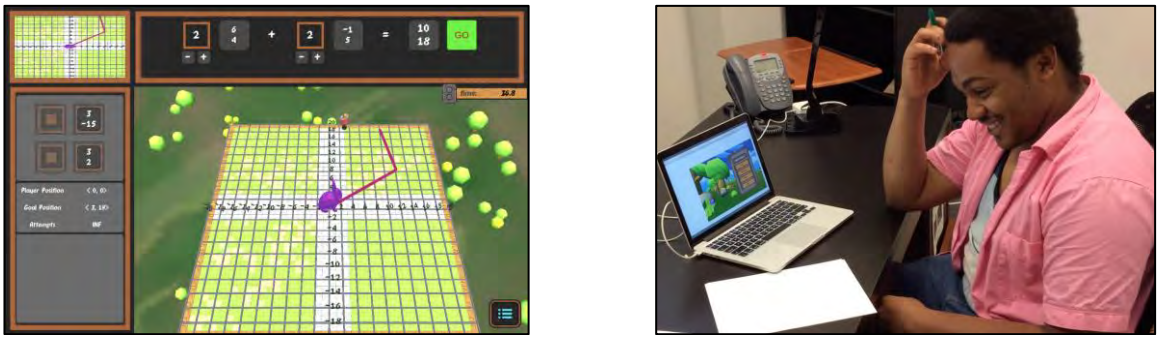


Figure 2: Screenshots of growing figures from Ellis' presentation

Following this, Ellis had group participants share and discuss and then shared out their experiences among the whole group. Ellis then presented brief examples of student data that exemplified some of the phenomena that the group had experienced. Following this presentation, Ellis outlined her team's working definition for playful math as instances in which students engage in activity with (a) agency in exploration, (b) self-selection of goals, (c) self-direction in how to accomplish the goals. The ensuing discussion addressed several topics, notably the importance of focusing on student agency and the role it plays in authentic play as well as the tension between structure (toward an intended learning goal) and autonomy (the natural progression of the player's activity).

Plaxco then presented examples from his research collaboration in designing and creating the videogame *Vector Unknown* (Figure 3a; Mauntel, Levine, Plaxco, & Zandieh, 2021) for supporting students' Linear Algebra understanding that is based on the existing Inquiry-Oriented Linear Algebra curriculum [IOLA; Wawro, Zandieh, Rasmussen, & Andrews-Larson, 2013]. In his presentation, Plaxco provided an additional instance of the importance of layer agency by describing a player (Lance, Figure 3b) deciding to play what Lance described as a "meta" game of achieving an *additional* goal beyond the goal of the game intended by the design team. The

resulting discussion connected Ellis' presentation with Plaxco's, specifically focusing on the role that level design can have in shifting players' focus toward more authentic activities that can also tend to lead to richer mathematical discussions.

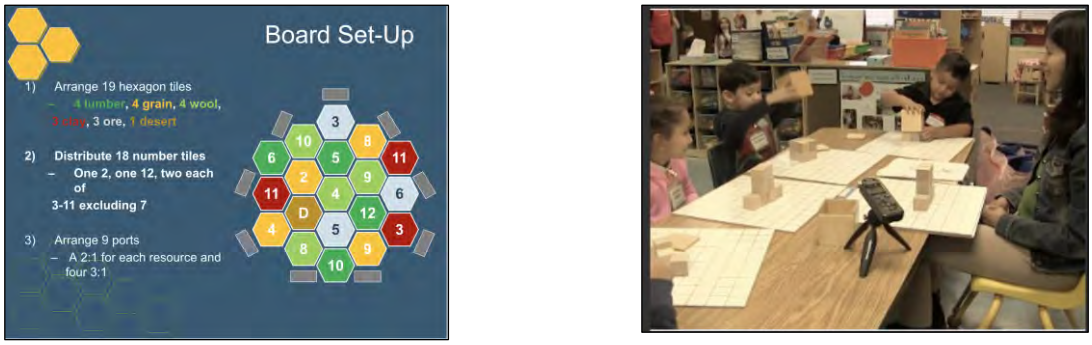


(a) (b)

Figure 3: A screenshot of the *Vector Unknown* (a) and photo of Lance deciding to play a "meta" game within the videogame (b)

Day 2: Adapting Play Activities to Support Mathematics Learning

On the second day, Molitoris-Miller, Reimer, and Simpson presented playful contexts that provide opportunities for mathematical thinking and learning. Molitoris-Miller suggested ways undergraduates encountered moments of mathematical play through explorations with the board game Catan (Austin, Kronenthal, & Miller, 2021). In particular, discussions focused on how board and game setup, players' strategies and solution methods, and students' mathematical arguments traversed the traditional boundaries of play and mathematics.



(a) (b)

Figure 4: Molitoris-Miller's described board setup for the game Catan (a) and Reimer shared analysis of guided play episodes in preschool classrooms (b).

Reimer then presented his research conducted in preschool classrooms to explore how young children and teachers engaged in spatial reasoning through play (Reimer, 2021). Reimer described guided play episodes that encouraged children to explore a variety of contexts and materials. In particular, discussion focused on ways teachers and children participated in play together, with specific attention to the agency and authorship afforded to children through co-participation in play.

Finally, Simpson engaged the working group in a tinkering task (Simpson, Zhong, & Maltese, 2022) using common materials to design a delivery mechanism for sharing objects with a friend next door. The group used popsicle sticks, elastic bands, straws, and other materials to

construct their mechanisms. Simpson guided discussions around the potential of low-notation, playful environments to provide opportunities for youth to experience, express, and build upon mathematical practices in a context that is not bounded by standardized tests and textbooks.

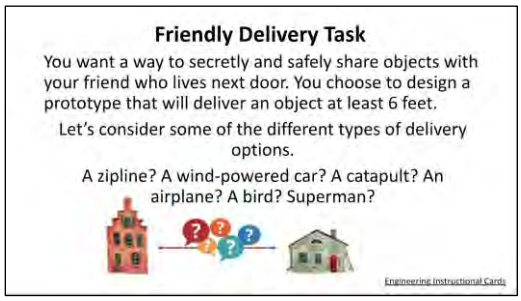


Figure 5: Simpson's engineering task offered opportunities to legitimize informal ways of thinking about mathematics.

Day 3: Intra-working-group Discussion with EMIC

The final day, the Mathematical Play and EMIC working groups met together for a session focused on exploring areas of overlapping interest. In particular, the two working groups converged to explore play as an embodied approach to mathematical learning, and embodied activities as opportunities for mathematical play. EMIC group members, led by Hortensia Soto, guided participants through an activity focused on supporting an embodied sense of the geometric properties of triangles, specifically, the fact that the interior angles of any triangle sum to 180° and external angles of any polygon sum to 360° (Soto, 2021). The groups also discussed how this activity might be generalized for any closed polygon. The following discussion centered around the importance of embodied activities for mathematical learning and discussed potential commonalities between the play, embodied mathematics, and creativity.

Ongoing and Future Work

Since the conference, several subsets of attendees have continued collaborations and intend to develop these into extensive projects. For example, co-author Ellis and meeting attendee Rob Ely have secured internal grants at each of their universities, have continued to analyze their play data, and have developed analytical framings for mathematical play, which they intend to present at the next PME-NA conference. Co-authors Reimer, Williams-Pierce, and Simpson have continued collaborations, meeting to develop their focus on failure and feedback in free-play environments. Co-authors Plaxco, Zandieh, and Mauntel have continued development of a 3-dimensional version of *Vector Unknown* and are collecting data with that version of the game during Spring 2022. In reflecting on the Working Group, co-authors Plaxco, Reimer, Williams-Pierce, and Ellis identified agency as a critical aspect of mathematical play that warrants further development. This aspect of players' experiences emerged across every discussion and presentation during the working group session as a critical component for not only successful play, but also deeper mathematical meaning to emerge during play. In light of this, we have chosen this as the central focus of the next Working Group, which we intend to shift in format from years' past. Specifically, with our next Working Group, we intend to spend the time collaboratively developing theoretical framings related to Mathematical Play. The primary goal of this work is to develop a collective touchstone for: (1) defining mathematical play in a way that situates all of the group members' work within the field, (2) identifying and characterizing

the role of agency in mathematical play, and (3) identifying meaningful structural connections between mathematical practices and playful disposition.

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