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ABSTRACT

Given the recent attention on early childhood mathematics education, it is important to understand how early childhood educators conceptualize the work of mathematics teaching and learning. This article describes a study of preschool educators' conceptions of mathematics teaching and learning in the context of a multi-year professional development project at two Head Start preschool centers. In particular, I examined how participants in this study perceived their roles in mathematics teaching and learning, and how their conceptions demonstrated awareness of children's ways of thinking and learning in play. I found that educators' conceptions converged around three themes of practice: 1) engaging and nurturing, 2) noticing children's mathematical activity, and 3) guiding children's mathematical learning. These conceptions shed new light on the promises and challenges of professional development aimed at improving mathematics teaching and learning in early childhood education.

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Early childhood mathematics has recently received increased attention through research that reveals the importance of early mathematical development (NAEYC & NCTM, 2002; NRC, 2009). Analysis of longitudinal data suggests that early mathematics influences later development in mathematics and literacy (Duncan et al., 2007; Nguyen et al., 2016). Recent findings have also shown that young children engage in more mathematical thinking than was previously believed (Baroody, Lai, & Mix, 2005; Ginsburg, Lee, & Boyd, 2008). Early childhood teachers play an important role in young children's mathematical development through their planning of mathematical experiences and interactions with children (Ginsburg et al., 2008). However, these educators do not typically receive the same preparation as do K-12 teachers (Ginsburg et al., 2008); hence, they may be underprepared to take advantage of learning opportunities during mathematics experiences in the preschool classroom.

In addition to high quality teacher education, professional development (PD) plays a role in addressing this dilemma. While the educational field has evidence-based best practices for professional development (Desimone & Garet, 2015; Fishman, Davis, & Chan, 2014; Kennedy, 2016), programs based on these practices have not always led to improved mathematics learning opportunities in early childhood classrooms (Piasta, Logan, Pelatti, Capps, & Petrill, 2015). Particularly troubling to the field of early childhood mathematics is the lack of professional development efforts to support early childhood educators, despite

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the recognized importance of children's early mathematical development (Schoenfeld & Stipek, 2011; Simpson & Linder, 2014). Those who work with in-service and pre-service early childhood teachers have acknowledged a need for further clarity regarding mathematics content expectations and pedagogical approaches for early mathematics instruction (Parks & Wager, 2015; Whyte, Stein, Kim, Jou, & Coburn, 2018). Thus, providers of professional development and teacher educators alike are tasked with the challenge of better understanding the nature of early childhood mathematics teaching and learning.

These challenges are mirrored with an existing gap in the literature related to early childhood mathematics. According to a review by Parks and Wager (2015), only 12% of articles published between 1994 and 2014 in prominent educational journals focused on early childhood mathematics. Moreover, they found that 94% of articles published in early childhood journals were not focused on mathematics. Researchers in early childhood mathematics education, however, are giving attention to pedagogical approaches that are suited to early childhood mathematics instruction and are making recommendations for educators. *Guided play*, for example, is a play-based pedagogical orientation that presents an alternative to more direct forms of teaching (Weisberg, Hirsh-Pasek, & Golinkoff, 2013). This practice requires early childhood teachers to reconsider conceptions of their roles in classroom mathematics instruction and encourages teachers to develop playful learning experiences that are both child-oriented and teacher-scaffolded. Approaches like guided play require pedagogical shifts on the part of early childhood teachers that have implications for the planning and delivery of teacher education and professional development.

Considering this paradox of promise and challenge, further efforts are needed to understand how early childhood educators approach mathematics education. I sought to contribute to these efforts by examining how a cohort of Head Start preschool educators conceptualized the work of mathematics teaching and learning at the outset of a professional development program focused on play-oriented mathematics pedagogy. Through analysis of interviews and classroom observations, the study explored educators' enactment of their roles and practices as they began the professional development. The following sections present a brief overview of relevant research and theory on teachers' conceptions, practices, and roles related to teaching mathematics in early childhood.

Teaching early childhood mathematics

Early childhood teachers' knowledge of children's mathematical development plays an important role in informing teachers' decisions in the preschool classroom. Specifically, teachers' knowledge about how children develop mathematical ways of thinking and acting guide learning interactions (Ginsburg et al., 2008; Schoenfeld & Stipek, 2011). For preschool teachers, this knowledge typically includes an awareness of the progression of early understandings (e.g., the development of the number word sequence, counting, cardinality) and the ways children might make progress toward richer and more well-developed understandings (Clements & Sarama, 2004; Fuson, Richards, & Briars, 1982; Ginsburg et al., 2008; Steffe & Cobb, 1988). Knowledge for preschool mathematics teaching is a specialized form of pedagogical content knowledge (PCK; Shulman, 1986) that is distinctly connected to both the mathematical domain and children's development of mathematical processes. Preschool mathematics teaching, moreover, places a critical emphasis on the active participation of teachers with

children: "Following the thinking of children as they interact with materials, recognizing the mathematical potential in their activities, and knowing how to comment on and extend their mathematics-related thinking all must be central" (McCray & Chen, 2012, p. 297). Through her study of preschool teachers, Lee (2017) articulated three activities that characterize the enactment of pedagogical content knowledge in preschool teaching:

> (1) awareness of opportunities for mathematical learning in children's play; (2) interpreting children's math activities based on preschool mathematical concepts; and (3) enhancing children's in-depth mathematical thinking. (p. 231)

Framed in this way, preschool mathematics teaching relies on how teachers interact with children. In Lee's words, "Teachers' knowledge is situated in their actual practice" (p. 231). Given the nature of preschool mathematics teaching and the interactive role of the teacher, it is important to understand how preschool teachers conceptualize mathematics teaching and learning, how they make use of children's thinking, and the roles they embody in classroom interactions.

Preschool teachers' conceptions of teaching and learning mathematics

Preschool teachers' views of teaching and children's mathematical learning inform their practices. Teachers enact conceptions about how children develop knowledge and understanding, the types of activities classroom interaction should facilitate, and the role of child-initiated experiences in exploration and play (Whyte et al., 2018). As an example, if a preschool teacher believes mathematics is a body of knowledge that requires careful modeling and step-by-step instruction, directive teaching interactions and an emphasis on rote skills will likely dominate classroom activity. On the other hand, if a teacher views mathematics as a way of organizing and sense-making, classroom activities are more likely to support students in meaning-making opportunities.

Although previous studies have suggested that preschool teachers have prioritized language and literacy over mathematics and have considered mathematics a difficult subject to teach (Copley, 2004), recent findings suggest preschool teachers are considering new perspectives on early childhood mathematics. In their survey of 346 preschool teachers, Chen and colleagues (Chen, McCray, Adams, & Leow, 2014) found that a majority of teachers believed that mathematics education was appropriate for preschoolers, with 87.6% of participants reporting that children learn about mathematics through everyday experiences. They also found, however, that 62.1% of teachers believed that most preschool children entered their classes with minimal mathematics knowledge, and just over half of the surveyed teachers reported knowing what children knew about mathematics when they entered preschool. Although these findings indicate mathematics instruction may be an increasing priority, it is clear that preschool teachers face challenges in noticing and building on the mathematics capabilities of their children.

In developing the Mathematical Development Beliefs Survey (MDBS), Platas (2015) examined pre- and in-service early childhood teachers' conceptions about teaching mathematics, their own roles in the classroom, and their views of mathematics learning and learners. Platas developed four categories of analysis:

> (1) age-appropriateness of mathematics instruction, (2) classroom locus of the generation of mathematical knowledge (i.e. teacher vs child), (3) socio-emotional versus academic (specifically mathematics) development as primary goals of preschool education, and (4) teacher confidence in mathematics instruction. (p. 297)

One particular theme focused on the "classroom locus of the generation of mathematical knowledge (i.e., teacher vs child)" and involved teachers' conceptions of where, with whom, and how mathematical learning occurs. Platas found that pre-service teachers and teachers at the beginning of their careers in early childhood education were more likely to place the locus of control with the teacher, whereas in-service teachers with two or more years of experience and further education (master's degree or math development courses) placed the locus directly in the middle of teacher and child. These differences affect the opportunities for mathematics learning teachers provide children. As Platas (2015) explained, "teachers' beliefs about who is responsible for children's learning of mathematics are related to how teachers provide support for learning in the classroom" (p. 298). Unfortunately, the lack of opportunities for early childhood educators to deepen their understanding of effective practices in early childhood mathematics (Simpson & Linder, 2014) can create a barrier to teachers' growth and development.

When considering early childhood mathematics instructional practices, preschool teachers must continually navigate their own roles in children's learning. Studies have suggested that teachers' self-positioning influences the structure of mathematical interactions in their classrooms (Graue et al., 2015). This is important to consider, given recommendations for early mathematics pedagogies such as guided play that exists between direct instruction and free play (Weisberg et al., 2013; Weisberg, Hirsh-Pasek, Golinkoff, Kittredge, & Klahr, 2016), development of classroom interest areas that support exploration and learning (Wager, 2013), and interactions with children during play (Wager & Parks, 2014). In addition, a variety of roles can be assumed depending on immediate contexts and needs of children. For example, a teacher might interrupt a content-oriented teaching moment to comfort a child or redirect an undesirable behavior.

Accountability in early childhood education contexts

Head Start preschool educators – the focus of the PD effort in which this study is situated – work in a shifting context of policies, practices, and values. Early childhood educational contexts have historically focused on children's social, emotional, and physical domains of development, school readiness, and early literacy, language, and numeracy. While these broad priorities have remained, Head Start preschool programs, in particular, have faced growing accountability measures linked to performance standards and student outcomes (Walter & Lippard, 2017). These measures reflect an increasing emphasis on academic learning and have pressed on teachers' conceptions of appropriate teaching and learning practices. For example, increased accountability for student outcomes might influence teachers' choices to utilize instructional methods which seem to be more "efficient" than others, such as direct teaching. Recent years have seen an increase in teacher education levels in Head Start teaching staff that have been shown to correlate with developmentally appropriate beliefs about teaching and learning (Walter & Lippard, 2017). Accountability

practices and curricular materials, however, have reflected pressures to meet performance standards by prescribing teaching techniques and prioritizing efficiency over innovation (Parks & Bridges-Rhoads, 2012).

Given the challenges of implementing mathematics education for young children that is uniquely contextualized in the environments of early education, it is important to consider the ways preschool teachers in specific contexts conceptualize and practice mathematics teaching and learning. Thus, in the context of a professional development program, I addressed the question, *how do participating Head Start educators conceptualize children's mathematics learning and their own roles in supporting children's mathematical development?*

Method

This study was grounded in the situative perspective that considers the role of interaction and participation in physical, social, and cultural contexts in knowing and learning (Brown, Collins, & Duguid, 1989; Greeno, 1998; Lave & Wenger, 1991). Across the variations of the situative perspective, learning is considered to be a situated, social, and distributed activity (Putnam & Borko, 2000). In considering the mutually influential relationship between individuals and their environments, I took on Sfard's (1998) perspective of knowing as action rather than knowledge as product: "the permanence of having gives way to the constant flux of doing" (p. 6). Similarly, Lave and Wenger (1991) pointed to participation as "activity in and with the world" and on the premise that "agent, activity, and world mutually constitute each other" (p. 33). Taken together, these perspectives shaped how this study sought to explore the phenomenon of teachers' conceptions of mathematics teaching and learning. The resulting emphasis on knowing as action guided this study's exploration of teachers' described and enacted behaviors, actions, and practices.

Based on this grounding, this study employed a phenomenological methodology to develop descriptive accounts of educators' conceptions of teaching and learning mathematics in preschool. I used thematic analysis to identify, refine, and summarize patterns observed within and across participants. Through analysis of educators' conversations and interactions with children in the classroom, I sought to "identify tensions and distinctions" that might provide a more nuanced understanding of educators' enacted conceptions (Glesne, 2015, p. 184). The ultimate goal was to provide a multilayered account of the themes and variations within educators' conceptions of teaching and learning mathematics, with a particular focus on the practices through which educators enacted their own roles to support children's learning and mathematical development.

Professional development program with Head Start educators

A small research team and I engaged in a year-long professional learning partnership with a cohort of Head Start preschool educators to improve mathematics teaching and learning. The goal of the program was to explore playful environments, activities, and interactions that would support children's mathematical development. The research team was committed to disrupting traditional forms and deliveries of professional development that treat participants as receptors of specialized knowledge or prescribed strategies. Instead, we engaged with educators as co-participants in expanding collective notions of what might

be possible in preschool mathematics. Each member of the research team had experience in teaching and coaching, and had master's degrees in mathematics or STEM education.

Professional learning sessions occurred twice a month throughout the school year. A focus of the program centered on developing awareness of mathematical learning opportunities through interactions with children in play. Each session began with a collaborative and interactive experience that engaged participants as learners. For example, in one session participants stretched colorful flagging tape in small groups to create three-dimensional figures between them. The research team also routinely presented video-recorded activities and interviews with the children in participants' classrooms as a way of deepening their understanding of children's thinking (Reimer, 2017). Then through guided reflection, participants considered ways to support children's mathematical development through playful activities such as block play, interactions, and teaching practices. To support educators in the context of their daily work, the program also included classroom coaching and curriculum planning.

Context and participants

This study was conducted at two Head Start preschool centers in central California. The two centers were selected based on previously established relationships, willingness to participate, and administrative support. The 25 participating staff members included teachers, teacher-assistants, and center directors. Participants did not receive compensation for participation and all activities occurred during work hours and were approved by Head Start administration to satisfy teachers' professional development requirements. Professional learning activities took place during non-student days in one of the preschool classrooms. Interviews and observations of classroom practice took place during the normal school day. One site had nine female staff members and the second site had 16 female staff members – together the two sites served approximately 200 three- and four-year-old children. Among the participating staff members were 22 Latinx women, two Asian women, and one Black woman. Staff members ranged in length of teaching service from being newly hired to having over 20 years of experience in education. For this analysis I focused on a subset of 10 teaching staff members representing a range of perspectives and practices (see Table 1).

The local Head Start programs in this study served neighborhoods and communities comprised largely of Latinx and Black populations. As a White male researcher, I was aware of my inability to fully interpret and make sense of participants' experiences. My whiteness privileged me toward a mainstream narrative that included numerous opportunities for education, career advancement, and research. I was less acquainted with the counter-narratives Head Start educators experienced and risked demonstrating an insensitivity to the social and cultural context in which educators worked (Milner, 2007). Thus, I relied on the contributions and interpretations of my research team. The research team comprised one white female, two Latinx females, and one Asian female. The team assisted in cultivating honest and open conversations throughout interviews and observations. Throughout the study I held an awareness of my own position and the inherent power dynamics as I engaged educators in talking about their conceptions and practices.

Table 1. Background information shared by participants.

Participant (pseudonym)	Years of Experience	Background
Vanessa	17 years	Vanessa began as a disability assistant, then worked as a teacher's assistant, and is now a teacher working with three- and four-year-olds.
Sarah	17 years	Sarah has been a teacher assistant in a Kindergarten classroom, a Head Start teacher assistant for two years, and a preschool teacher for 15 years.
Ashley	14 years	Ashley's first two children were in Head Start. "I really learned a lot from those teachers and thought, I want to use those skills with my children."
Ella	24 years	Ella volunteered for Head Start when she was 20 years old. She began her career with Head Start in 1993 as a teacher assistant. She earned her degree and became a teacher in 2004.
Maria	26 years	Maria was in high school when she went to work in the Migrant Preschool Program to clean restrooms. After funds ended she was hired by Head Start in her junior year of high school.
Nancy	1 year	Nancy is still in school getting her A.S. degree. She first learned about Head Start when her daughter was four years old. She volunteered as a parent and liked it, so she stayed in it.
Sonya	Not provided	Sonya was working in a school where she got to know the teacher of the Head Start class. The teacher encouraged her to go to school with her to get university credits, and after helping in the classroom, Sonya joined the Head Start staff.
Elena	20 years	Elena has her B.A. from the local university and started as a substitute in schools.
Vero	25 years	Vero received a parent certificate for participating in her child's Head Start classroom. She did not have any schooling, but she started as a teacher's assistant.
Janet	17 years	Janet has lived in the area for 40 years and has been employed by Head Start for the past 17 years.

Data collection

We collected data that would help us form understandings of participants' conceptualizations and experiences in early mathematics teaching and learning. Throughout the professional development program, the research team interviewed Head Start educators, video-recorded professional learning sessions, conducted stimulated recall interviews, and kept field notes from classroom observations.

Interviews

Research team members conducted semi-structured interviews with participants. These audio-recorded interviews occurred at the beginning, middle, and end of the school year. Each interview was conducted by one of the four female research associates to establish an overview and background of participants' experiences and orientations toward mathematics teaching and learning. Participants were asked to discuss examples of their mathematics teaching practice and their views of children's learning. Because we were interested in how educators conceptualized mathematics teaching and learning, we asked interview questions such as: *Can you describe some of the activities that might be going on in your classroom that involve math?* These interviews helped to build trust with participants and developed a mutual understanding of the evolving learning partnership.

Video recordings of professional learning sessions

The professional learning sessions were video-recorded by the research team. When possible, we used additional audio recording in small groups to capture participants' conversations and reflections. We used recordings of these sessions to examine educators' participation and inform future sessions and classroom observations.

Stimulated recall interviews

As another way of accessing educators' meaning-making processes, the team conducted stimulated recall interviews with participants several times throughout the school year. Classroom activities were video-recorded and replayed with the participants as an opportunity for them to reflect and comment on the interactions. These conversations generated insights about educators' roles in classroom mathematics, the conceptions guiding their interactions with children, and children's mathematical activity in classrooms.

Classroom observations

Team members conducted classroom observations and coaching visits on a weekly basis. The research team kept anecdotal field notes and developed researcher memos from these visits. These artifacts supported interpretations of how educators interacted with children and made adaptations in teaching practices. They also helped to guide the ongoing planning and revision of the professional development.

Data analysis

Data analysis began with a look across all collected data from the first two months of the program to establish an initial sense of educators' conceptions about mathematics teaching and learning. The research team met weekly to discuss observations from the variety of data and considered interview data alongside observations of practice to cross-check the developing interpretations. After a review of the transcripts from the initial participant interviews, two areas of interest emerged: (a) the teacher's role in mathematics activity and (b) conceptions of children's learning. I used a categorical coding matrix (Maxwell, 2013) to organize excerpts from interviews and stimulated recall, professional learning sessions, and field notes from classroom visits. I then used thematic analysis to group responses around similar themes and explore the variance within themes. For example, excerpts from stimulated recall interviews that were relevant to established themes were transcribed, coded, and entered into the matrix. Ongoing interpretation resulted in the development of descriptive subcategories for each theme. Key words and phrases were generated in these descriptive subcategories to identify emerging patterns.

I organized data on educators' roles during mathematics activity around emergent codes that served as broad descriptors: *engaging and nurturing, noticing, adapting*, and *guiding*. These and other terms were not used to refer to existing constructs in the literature. For example, professional "noticing" has been used to describe teachers' spontaneous decision-making based on children's understandings (Jacobs, Lamb, & Philipp, 2010). In contrast, I developed ongoing, broad meanings for each of these categories by generating key words and phrases such as *helping, guiding, showing how, intervening*, or *supporting* into sub-codes based on analysis of interview data and observations. Data organized around the theme of children's mathematical learning centered on children's mathematical activity, the materials and activities children used, and educators' ideas about children's learning. Descriptive subcategories provided further clarification to the multiple views of children's learning that the cohort of participants held, and included the following: *counting and cardinality, learning, exploring, participating*, and *developing skills*. For this paper, I have organized the findings around three broad themes: engaging and nurturing, noticing children's mathematical activity, and guiding children's mathematical learning (see Table 2).

Table 2. Themes, sub-codes, and descriptions.

Theme	Sub-codes	Description
Engaging and Nurturing	caring, ensuring safety, being a role model, leading	This theme represents the variety of ways teachers viewed their roles related to nurture and caregiving.
Noticing Children's Mathematical Activity	counting, learning, exploring, participating, developing skills	This theme comprises the variety of children's behaviors, actions, and conceptions that teachers noticed during mathematics activity in the classroom.
Guiding Children's Mathematical Learning	helping, guiding, showing how, facilitating, intervening	This theme describes the ways teachers interacted with children during mathematical activity and their roles related to children's mathematics learning.

Findings

Overall, data analysis revealed that participants held similar perspectives on mathematics teaching and learning. Some variations were seen in the roles educators enacted in their interactions with children. Consistent with the situative perspective guiding this study, I attended to educators' conceptualizations in the context of their teaching and interaction with children. Although not all participants were teachers, I have used the term "teachers" to represent all participants because they took on roles that shaped teaching and learning in classroom activities.

Engaging and nurturing

Teachers recognized young children's need for nurturing relationships and expressed a commitment to interacting with children to support them socially and emotionally. All teachers mentioned their roles in providing social and emotional support, often describing their efforts during mathematics activities to help children develop confidence, learn to share, and gain independence. Ashley described how she took on the role of caregiver: "My role is to make sure the kids are safe. That's my main concern. That they're safe and happy and healthy so they can learn. I see them as little people that need to be respected, too." As Vanessa explained, "Sometimes I'm a mommy. I had to hold a child and rub his back and calm him down." When asked about what she found to be challenging in her role as a preschool teacher, Vanessa elaborated:

> When you're trying to teach 18 children and you have two who are not ready to sit, who are not ready to learn. As a teacher it's your role to go and talk to those children and engage them and bring them in. And sometimes that's hard because I might engage them and bring them in, but then I've lost the others. So that's the hardest for me. (Vanessa, initial interview)

Here, Vanessa articulated a theme that was present across all teachers' perspectives: nurturing actions are key to engaging children in learning opportunities. Vanessa's perspective also reveals a commonly held notion that some children may not be ready to learn. Vanessa's explanation suggests that behaviors such as sitting on the carpet and listening are required for preschool learning. In one sense, these practices may be what teachers have put in place due to increased academic demands. Other teachers described their efforts as largely focused on preparing children for kindergarten. To this end, teachers were committed to helping children develop classroom behaviors that would be expected of them in kindergarten. This left me wondering if teachers had examined whether these same expectations were appropriate for preschool children.

When describing her classroom practice, Sarah expressed a similar commitment to nurture, but highlighted her engagement with children: "I engage in activities with them. They need to feel confident that we will be there for them. Especially the ones who need more help – I go and work one-to-one with them." When asked about her role in the classroom, Ella, a veteran teacher, described her perspective related to children's learning:

> It's not all academics. If a child cannot get along with other kids or sit long enough to hear instruction, or know how to cope with frustration or disappointment, school is going to be hard all around. They can know their names, their numbers, how to add or subtract, but that's not going to get them through school. So my philosophy has always been to meet their social and emotional needs. Because if a child feels safe in their environment and that they are confident and cared for, that's when the learning takes place. They feel comfortable when they know they are ok and here to learn. When the stress levels are high then they can't learn. For the children who cry all day, who's going to learn while they're crying? So you have to be able to get to them and make them feel comfortable. (Ella, initial interview)

Ella's classroom practices reflected her commitment to support children's social and emotional needs to help them learn. In one conversation following her interactions with a child during a mathematics activity, Ella wondered whether the challenges the child faced might have been compounded by her own difficulty in understanding the child: "I think it would be interesting to know what he knows – how much it might be language, us understanding him." Given the increased academic pressure faced by many preschool programs, Ella's practice of nurturing resonates with a historical commitment to social and emotional development in the field of early childhood education. Her concern for the dispositions children develop while in her care was grounded in her desire for them to learn. Her views of children's learning, similar to Vanessa's, reveal a conception that children's learning opportunities depend in part on their ability to "sit long enough to hear instruction".

Veteran teacher Maria also commented on children's engagement, emphasizing the negative effects of pressure on her children's willingness to participate. Her perspective suggested several additional elements of engagement: children's alertness, imagination, and participation:

> Right now it's making it fun for them. I know if you pressure them, the less they want to do it. They don't know that they're learning math, but they are. Making sure that they're kept busy and entertained. I look for the growth, how alert they are. Their imagination. How they're involved with all of the areas of the classroom. (Maria, initial interview)

Overall, teachers shared a belief in the interrelatedness of nurture and engagement in classroom experiences. In most cases, their practices revealed they believed that learning opportunities for children existed when children demonstrated behaviors indicating they were ready to learn. Teachers recognized their interactions with children had the potential to afford valuable learning experiences. In essence, these practices demonstrated a commitment to nurture in the service of learning.

Noticing children's mathematical activity

Teachers engaged children in mathematical activities in one-to-one, small-group, and whole-group settings. Their practices demonstrated a noticing of children's behaviors during these activities. Overall, teachers' noticing of children's mathematical activity was

shaped by the contexts in which it was noticed. In conversations about mathematics opportunities, they acknowledged the differences they noticed in children's mathematical development. In particular, teachers often discussed how children engaged in counting activity. During a reflective conversation after a classroom center activity, Nancy articulated what she had noticed about one child's counting behaviors:

> She said the number and she showed the finger pattern and she was able to remember what she was counting. Like hold the number in her mind. "1, 2," and when asked how many, she could say "2". Yeah, I was surprised with that. Cause a lot of kids I see would start counting again. (Nancy, stimulated recall interview)

In this example, Nancy took an observational stance in her classroom; her comments revealed her perspective that something could be learned by watching children's mathematical activity. When asked how many they have counted, children often interpret the question of "How many?" as a request that items be counted. Developing a goal to determine the total number of items comes as a natural step in children's counting development. Nancy's surprise at this observation may indicate that she has not interacted with children who are making these connections, or that her expectations for this child were surpassed. This noticing on her part represented an opportunity for her to develop deeper pedagogical knowledge in the context of her observations – to learn from her interactions with children.

While recalling a specific teaching episode, Sonya described the ways she observed variation in children's mathematical activity. She observed that children counted in different ways, specifically referring to the development of their number word sequences. Her description of a classroom activity reveals her understanding that incorrect number word sequences can be quite stable and may require numerous counting experiences to develop correctly.

> Yes, we'll start from 1 with little blocks and even if the child before them has counted 1-2-3-4-5 another child will come in and say 1-2-4-7-8-9-10 even though there's five and they've heard the child count them. Sometimes I will get the same little five blocks and even if they're different colors I will move them around to try and get the other children to see that it's still five but in different order, to see if they come up with the five. And I will ask "Is it the same?" and some will say no and some will say yes. (Sonya, initial interview)

Sonya's comments suggested that since a child had observed another child counting accurately, repeating the counting sequence should have come more naturally. Although Sonya demonstrated attention to individual children's activity, she did not intentionally begin with children's competencies; rather, she focused on a particular task that was presented to all children. She recognized that her role as a teacher included a careful noticing of how children counted when presented with a particular task.

In contrast to Nancy and Sonya, Ella shared her perspective that children come into preschool with existing understandings. "Kids all come into this classroom knowing things already. It's our job to enhance that." She explained that part of her job was to find out where children were so she could meet their individual needs. Several other teachers shared this commitment, referencing the need to get to know children at the beginning of the year in order to explore children's levels and skills. Ella explained that her goals were to enhance children's understanding of numbers and to utilize more word problems. When discussing her observations after watching a teaching episode from her classroom that was video-

recorded, Ella reflected on the ways she noticed individual children's activities and expressed the challenges she faced in observing all the ways children were interacting in the classroom during the lesson:

> You miss so much in your class – like those things I just saw, I didn't see in the class. I was critiquing myself, what are some of the things I could do differently. Like what Jasmine was doing with patterns, even that child who answered you, she didn't do that at the carpet. Just watching them interact and play – you miss a lot of that because it only takes a second in the classroom. (Ella, stimulated recall interview)

Ella observed that noticing children's activity in play was challenging. During play, children are led by their own goals, make up their own rules, and often allow imagination equal footing with the real and tangible. Most noticing from teachers, however, occurred during structured activity such as whole-group rote counting, or when children were presented with specific tasks. For those teachers like Ella who expressed the importance of noticing children's existing mathematical knowledge, there was significant variation in the ways teachers' noticing of this knowledge impacted their interactions with children.

Guiding children's mathematical learning

In conversations about their mathematics teaching activities, teachers described their actions using a range of verbs such as *guiding, facilitating*, and *intervening*. Teachers varied in how they defined this guidance, as well as in the ways they interacted with children during mathematical activity. In guiding or facilitating learning, teachers planned intentional mathematics activities for children, involved children, and engaged in activities with them. Most teachers articulated a balance between teacher-directed instruction and following the child's lead. On one hand, teachers expressed the clear role they held in teaching and introducing children to important mathematical ideas. However, in many instances they also indicated that they routinely modified activities and materials based on individual children's needs and developmental levels. In this way, teachers conceptualized teaching and learning as responsive, adaptive activities that flexed based on children's levels and needs. For example, Elena described the way her views of teaching mathematics were dependent on time, context, materials, and learners:

> I do like to impart math to children as I teach them. I know that at the very beginning I can't do very many things yet until I know where the children are, because you know, you're not going to go out and ask, Do you know this or that? They won't know what you're talking about. Exploring is the first thing they need to do. They explore whatever you have in the classroom, the toys that you have, the things that you put out. (Elena, initial interview)

Here, Elena did not dismiss the instructive nature of her role as teacher. Yet, she understood that children's learning was largely dependent on opportunities to explore. In her classroom, Elena provided opportunities for exploration in various centers with materials as well as in daily activities like setting the table and mealtime. "Everything is an opportunity to have math in the class. There are so many different ways you can do math. You can't escape it." When discussing individual children, Elena's observations highlighted the skills she observed and the ways she modified activity based on her observations. "Some of the children do not come in counting to three. What I do with them is we just count. We count to ten with claps."

Elena elaborated on the idea of guiding children's learning by sharing a story about a time she brought a tomato plant into her preschool class. She showed the plant to students and asked them what they noticed about it. As children pointed out the various attributes and parts of the plant, Elena posed questions to children: Can you count the leaves? Can you count the tomatoes? She explained her perspective on the ways she interacted with children:

> There are so many ways that you can teach math to the children, it doesn't have to be sitting down with 1 and 2, and they're very inquisitive, but you have to kind of guide them to it. When you teach them you can let them find the experience and maybe you can guide them there intentionally. (Elena, initial interview)

Vero, a teacher with Head Start for 25 years, described the way she approached teaching children. "Sometimes I'm a kid myself. I try to get down to their level and talk to them, and be their teacher and help them in any way I can." Janet's teaching practices revealed a similar reliance on child-initiated activity while still maintaining expectations for learning. When describing the ways she interacted with children, Janet emphasized the facilitative nature of her role:

> It is amazing what children can create with materials such as playdough and blocks. I would say I'm a facilitator because I'm helping support them in their learning. Maybe setting out some things with my expectations in mind – say, I want them to create something I have in mind with playdough, connectors, or blocks. (Janet, initial interview)

Following a classroom observation, Janet described how she felt the math activity had gone. Her first comments were related to how the children attended to the activity. She noticed that as children waited for their turn, they seemed to lose interest in the activity. During the lesson, Janet asked one child a question related to the concept of more or less, and when asked to explain why she had chosen this concept, she replied, "He understands that a little bit more versus the other kids. In other activities like going outside and counting or when he's playing with something at the table, he's knowing who has more and who has less." In this episode, Janet's expectations were formed in part by the child's observed activity. Janet also followed the curriculum lesson plan and used the suggested questions, although she explained that she felt some flexibility in how the lesson was implemented. "You can change it a little bit if you want."

Sonya described her role with children during classroom math lessons as involving direction. "The younger ones, they don't really understand it yet … and so again, there's some direction in there. We know the teacher has to intervene in there … " During one classroom lesson, Sonya engaged children in completing a construction paper model of a character, helping them to cut out legs, arms, eyes, and other body parts while drawing their attention to the shapes and number of pieces needed. The children then glued the pieces onto the main body of the character. A sample completed character was available for the children to replicate. Sonya sat with the children at the table and interacted with them as they worked, asking questions such as: Does he have one mouth or two mouths? What shape is that? We had a circle and we cut in half? What is that called? Is it a circle, or crescent, or half a circle? What does he need to stand? He needs two ___? Sonya assumed a directive role in her style of questioning and the ways she guided children through the planned activity. In this activity, children engaged in following directions, providing non-verbal nods, and giving brief responses to the teacher. Children were given few

opportunities to form their own thoughts, as most of the teacher's questions required one-word answers. In an interview before the lesson, Sonya elaborated on her role in mathematics teaching:

> My role is to be the role model, to teach them, to introduce this stuff to them, and if they're having problems with it, to continue to work with them as much as I can, you know, get any opportunity to help them with that.

It was not evident that Sonya drew on her knowledge of children's existing understandings in her efforts to facilitate the activity. The specific goals of the lesson and the fixed outcome did not provide opportunities for mathematical reasoning in ways that might have revealed how children were thinking mathematically. In this sense, Sonya's facilitation related more to the task completion rather than to building on children's existing competencies. When asked about how she felt the lesson went, Sonya replied, "I think it went well. They more or less knew where the pieces went – there's no right or wrong way to do the activity. Even if they put the legs here [pointing to other part of body], that's ok. The goal of the activity is the shapes and how many."

In many ways, regardless of the specific approach taken, teachers expressed a commitment to taking an instrumental role in children's mathematical development. In discussing their perspectives, some acknowledged the importance of following the child's lead. Yet, the opportunities teachers provided children to demonstrate their thinking were not consistently anchored in children's experiences. In addition, they were less likely to describe their interactions with children during play as capable of enhancing learning opportunities for children.

Discussion

The educators in this study relied on a variety of resources to make instructional decisions, such as the curriculum, their informal observations, and the standard preschool expectations such as counting and working with shapes. It is less clear, however, what role children's mathematical thinking played in informing their interpretations and resulting interactions with children. Although the teachers noticed opportunities for mathematics in various classroom contexts, their observations of the children were bound by structured tasks and the specific competencies they expected to encounter. With priority on developing school readiness and achieving mandated child outcomes, it is possible that Head Start educators face a narrowing field of vision that minimizes opportunities to acknowledge and build on the wide variety of children's mathematical thinking. As Lee (2017) suggested, the teaching practice of interpreting children's activity requires both a willingness to follow the child's lead and a knowledge of children's mathematical development. This study's findings point to the need for interactions wherein Head Start teachers can both guide and learn from children.

This study also suggests several nuances to Platas' (2015) description of mathematical development in preschool that is "child-initiated and teacher-supported" (p. 306). Across the variety of conceptions and roles represented in this study, it is clear that participation between teacher and student must be carefully negotiated. The space that teachers create for mathematical exploration holds potential for shared interaction and learning, "a space for joint action" (Davis, 1996, p. 269). The interplay between teacher and student in joint action

repositions the locus of the generation of mathematical knowledge into *interactions* rather than *individuals* – knowing that can be enacted in interaction between students, and between students and teacher. From a theoretical perspective, the findings of this study suggest that dichotomizing conversations that position teacher-centered mathematics in opposition to student-centered mathematics do not provide fruitful space for considering the possibilities when teacher and student participate together in meaning-making. This study suggests that early childhood classrooms are particularly well-suited to explore possibilities for knowing in the interaction spaces between children and adults – to "create those openings where we can interactively and jointly move toward deeper understandings of a shared situation" (Davis, 1996, p. 239). This approach makes use of the cultural and material resources that both teachers and young children bring into the space and represents a way of developing mathematical knowing that disrupts traditional dichotomies of teacher and learner, known and unknown, expert and novice. Such mutual participation requires interpretive knowing on the part of early childhood teachers that can inform their decision-making processes during mathematics activity.

Implications for professional development in early childhood mathematics

This study suggests several implications for professional development in early childhood mathematics. First, as seen in this study, educators have complex views of children's mathematics learning that are influenced by the values and expectations of schools, cultures, and communities. Professional developers should help educators examine their own interpretations of children's mathematics as well as expected learning outcomes. A recognition of the mathematical ways of knowing that children possess can help educators position children as natural learners and implement practices based on children's interests and assets, rather than on expected behaviors.

Another implication for professional development derives from educators' perceived challenge in noticing children's mathematical activity in play. As Ella articulated, the variation that exists in what children do according to the context of their activity posed a unique challenge. Professional development should focus on the mathematics opportunities teachers might observe and enhance during both structured activities and children's play in the classroom. In response to this challenge, as Parks and Wager (2015) suggested, teachers need more examples of mathematical noticing in early childhood contexts. Professional development efforts should provide a variety of these examples, paired with pedagogical practices that highlight the learning opportunities in both formal and informal activity. These efforts can help to resolve the perceived incompatibility of teacher-led instruction and child-driven activity.

Finally, teachers need time to consider how they can participate in playful ways to support children's developing mathematical thinking. An important step into this work is to help teachers embody the elements of play, and indeed, find ways to play for themselves that contribute to children's play. As suggested in the guided play framework (Weisberg et al., 2013), classroom mathematics might involve teachers co-playing with children, taking up children's ways of playing while carefully proposing new ideas and directions. Classroom coaching is well-suited to helping early childhood teachers consider new ways of being in their classrooms, new roles, and new patterns of interaction that may not align with their histories of interaction. Knowing more

about educators' existing conceptions, practices, and roles can help providers of professional development and coaches contextualize their efforts and better inform the design and development of experiences that engage teachers in exploring new roles and positioning related to both mathematics and children's learning.

Limitations

This study faced several limitations that warrant acknowledgment. First, the convenience sampling method yielded results that may not necessarily represent the broader population of early childhood educators. Head Start programs vary widely from site to site, and are one of many that serve young children. Educators in different regions or in other programs such as state-funded preschools or private programs may hold differ perspectives. Second, although pairing interview data with classroom observation was a strength in this study, the interpretations developed are still subject to the researcher's perspectives or biases. While member checking and routine conversations among the research team helped to minimize bias, the specific contexts in which classroom observations occurred and our interactions with participants shaped the ways we made sense of our findings. Finally, further study of the ways teachers' conceptualizations changed as a result of the professional development could provide greater insight into their roles and practices.

Conclusion

Early childhood educators play a significant role in young children's mathematical development. Professional learning programs must consider the types of experiences that can support preschool teachers in deepening their knowledge of children's mathematical thinking and ways to support children's learning in play contexts. Providers of professional development and teacher educators should explore ways teachers can open spaces for child-initiated mathematics learning throughout the preschool day. In preschool contexts faced with increasing academic pressure, attention to teachers' conceptions of their own roles and how these shape their interactions with children can help contextualize new approaches. Professional supports, such as coaching, can encourage teachers to take advantage of real-time opportunities for mathematical meaning-making with children, and through reflective conversation, can help teachers develop pedagogical orientations that guide their interactions with children.

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