

Designing for Productive Problem Posing in Informal STEM Spaces

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Abstract: This symposium offers a look across several sites of informal STEM learning for moments of productive problem-posing. Despite the importance of problem-posing in both academic and applied STEM fields, students are rarely offered robust opportunities to define their own paths through these domains. When they are, it is often in the context of informal spaces—museums or after-school clubs—and is frequently less well-defined. This symposium brings together three case studies—one of younger students in guided play and two of slightly older youth in object-directed play. By putting these in-depth analyses in conversation with one another, we hope to highlight important commonalities in order to distill possible design recommendations for creating more opportunities for productive, and creative, disciplinary engagement.

Overview of the symposium

The focus of this symposium is on the affordances of large-scale, collaborative, object-driven spaces for STEM-relevant problem-posing. In this symposium we attempt to bring together research on embodied cognition and the affordances of large-scale designs for disrupting traditional, classroom-based STEM tasks with work on informal learning environments that highlights the potential of student-defined, interest-driven engagement. We take as a starting point the idea that different environments make unique contributions to learning, and are best designed with an eye toward how the different pieces of a learning ecology might complement each other (Knutson & Crowley, 2001; Ito et al., 2013). Thus, informal learning environments are not meant to replace but instead to enhance and supplement more formal designs. In particular, we address the observation that formal, standards-driven, compulsory classrooms are well-tuned for breadth of conceptual coverage, but often miss opportunities to develop skill-sets and dispositions better approached through more ill-structured tasks (Lampert, 1990). At the same time, lack of detail about what it takes to design for these broader goals in ill-structured tasks often leads museums and other informal environments back to fun and engaging fact-delivery systems (Griffin, 1994; Olson, 1999).

In particular, skills-based K-12 education tends to focus on familiarizing students with prescribed standards and codified strategies such as standard algorithms (Stigler & Hiebert, 1999). Even museums sometimes revert to hands-on ways of delivering established information, such as “Can you guess?” flip charts and other kinds of simplistic interactive exhibits, often because the work of designing for “minds-on” as well as “hands-on” interaction is so complex (Allen, 2004). Advanced participation in math and science, however, involves not just executing on known problems, but actually posing novel, discipline-relevant problems and inventing strategies for solving them (Lakatos, 1976; Sinclair, 2004). And while progress is being made on these fronts in reform efforts like Project-Based Learning (e.g., Boaler, 1997; Boaler & Selling, 2017) and Teaching Mathematics for Social Justice (Gutstein, 2003), naturalistic studies in informal environments can offer a different vantage points for would-be designers.

Contribution of each paper to the broader discussion

Each of the case studies presented in this symposium offers a different perspective on, and context for, looking at problem-posing in interest-driven environments. From one angle, each paper represents a different case of students who invent and attempt to solve a novel problem that has potential relevance to STEM learning. Each environment is of course embedded in a set of cultural expectations, but in most cases participants are either explicitly encouraged to develop their own problems, or at least free from any negative consequences for doing something other than what the designer intended. From another angle, each of these circumstances has a strong set of both material and participatory constraints and affordances—one under the watchful eye of parents (paper one), one dictated by peer-defined social structures (paper two), and one by adult guidance (paper three)—that mediate emergent goals and solution paths.

We have two goals in bringing these papers into conversation with this symposium: 1) to present an existence proof that self-directed problem posing is not beyond the grasp of even young children, nor is it confined to a particular kind of setting, and 2) to look across sites with significant variation in an attempt to

distill broader design principles than might be warranted by a single small-scale study. To this end, we include several (possibly idealized) cases where youth (papers one and two) are essentially left alone with an object or objects and follow emergent goals that are more or less well defined, as well as one with younger children (paper three) in which facilitation was key. Finally, our goal in bringing these papers together is to highlight commonalities and differences across these cases in an effort to create humble theory about different kinds of material and social design choices for productive problem posing.

Discussant

To help facilitate this comparison, we also bring in Lauren Vogelstein, whose work on physical research in a dance company, and the affordances of large-scale object-mediated dance for developing mathematical intuitions and observations (Vogelstein, Brady, & Hall, 2017 & 2019), will offer another unique perspective on this work. In particular, her work on ensemble mathematical learning has shown how the embodiments of people-plus-prop systems, in which the manipulation of large-scale geometric figures necessitates performances by an ensemble, create a distributed problem posing environment. More specifically, the ensemble necessity of these set-ups requires full participation to animate ideas, which provides a rich physical substrate (Goodwin, 2017) for participants to manipulate. For example, a quartet of STEM educators choreographing a performance with a 7' x 7' silver Mylar sheet proposed ideas for coordinated ensemble performances with slight changes to what had previously been enacted by the ensemble in order to create new movements that made aesthetically pleasing manipulations of the square prop. In this example, the ensemble nature of the activity forced participants to make their ideas visible through partial enactments as they planned their choreography. Through this process the choreography became increasingly more complex, eventually surpassing their understandings as they performed a double reflection over both of the diagonal lines of symmetry of the square prop and were unable to replicate the choreography. These STEM educators came across an interesting problem through a process of aesthetically driven physical brainstorming, in which ideas were enacted as an ensemble, and the proposals to manipulate these actions became new forms of choreography. In her ethnographic work with a professional dance company, Vogelstein has also begun to articulate a similar process the dancers call “physical research” in which problems are iteratively posed by a choreographer and refined as dancers propose responses collectively with their bodies. Thus, she brings a perspective on ensemble learning (Ma & Hall, 2018; Ma 2017) to complement the other in- and out-of-school perspectives represented in the symposium.

Paper one: Manifesting mathematics: How playful engagement facilitates emergent problem finding

Lara Jasien

Overview and background

While school mathematics in the U.S. often can be adequately described as learning definitions and practicing procedures (Stigler & Hiebert, 1999), authentic engagement in STEM requires creative problem finding (Einstein & Infeld, 1938; Lakatos, 1976). For mathematicians, this kind of problem finding can be the product of playful exploration (Sinclair, 2004), where mathematicians tacitly ask themselves what is possible and what is interesting within a particular mathematical system. Yet, for many students, school mathematics feels like all work and no play (e.g., Boaler & Greeno, 2000; Jacobs, Lanza, Osgood, Eccles, & Wigfield, 2002), and indeed it is often organized for such work-like experiences (Jacobs, Hiebert, Givvin, Hollingsworth, Garnier, & Wearne, 2006; Litke, 2015). Yet, the distinction between work and play is pivotal for problem finding and solving: while work involves an individual structuring their activity to fit the environment (as in school), play involves the individual structuring the environment to suit their activity (Elkind, 2008). Thus, play environments provide an ideal context to examine children’s informal mathematical problem finding, as play allows children to exercise their agency in ways that shape what is possible. In this study, I examine children’s play at a mathematical playground that supports children to create patterns, designs, and representations with novel materials at four different exhibits, in search of moments where they explore what is possible and pursue activities in ways that they find interesting.

Theoretical framework

Mathematical activity in play looks much different than mathematical activity in school, as the tools, representations, and indeed, the problems look very different. Thus, accounting for mathematical activity in an empirical study of play requires methods typically not employed in accounts of school learning. This study is premised on the assumption that studies of mathematical learning should account for whatever participants

consider relevant as they engage in their own meaning-making processes (Jordan & Henderson, 1995). Methodologically and theoretically, this means that attention must be paid to human language, actions, and the representations that they produce, as these are all embodiments of mathematical activity (Hall & Nemirovsky, 2012). Thus, in this study, I examine how (and what kinds of) mathematics emerge through what DeFreitas and Sinclair (2014) call material entanglements—the interlacing of gestures, talk, emotion, and tools that co-constitute mathematical concepts. The products of these material entanglements, and thus of all mathematical activity, become a “site of agency” (p. 85), as they becomes tools to support future thinking.

In the kind of play under scrutiny in this study, this means that the patterns, designs, or representations that players produce carry with them traces of mathematical concepts that were co-constituted through human interaction with materials. Thus, I analyze how two 12 year old girls engaged with four different mathematical objects throughout their 36 minute stay at a mathematical playground. By attending to what the girls produced, their trajectory of attempting to use the materials for their own purposes, and their informal evaluations of their activity, I am able to make claims about the richness of their engagement in relation to the affordances of the mathematical playground, including but not limited to the design of the materials and the open participation structure where two pre-teens can explore together, set their own goals, and compare and evaluate their own activity.

Case study and preliminary findings

Notably, these two participants are part of a larger data corpus of 345 children between the ages of 4 and 16, with the majority of participants between 7 and 12 years old ($n = 279$). Analysis of these children has shown that the two focal participants’ activity is unique in how long they engaged at each exhibit (longer than typical), but not unique in the nature of the designs they produced in their play. Because their parallel play generated rich data, and because their engagement with the materials looks quite different from each other and thus represents a spectrum of engagement at the mathematical playground, they have been selected for a close analysis that gives rich understanding of the phenomenon (i.e., a revelatory case, Yin, 2003). The final paper will contain data to situate the focal participants within the larger corpus in relation to the patterns, designs, and representations they produce at each exhibit.

By attending to the multimodal engagement of two focal participants, I find that problem finding can be a generative process that leads to the emergence of mathematical meanings in play, even for children as old as 12 years of age. This analysis sheds light on the problems that children and adolescents can and do find for themselves when given opportunities rich and open enough to support meaningful engagement, and thus broadens what we can imagine as possible in mathematics education.

Paper two: “Our boat is magic”: Emergent social goals break open the black box

Katherine Chapman

Overview and background

The broader study from which this analysis derives focuses on a week-long sailing camp where students ages eight to eleven participated across two parallel learning sites: a living museum dedicated to maritime heritage, in which the students learned to rig and sail small boats in pairs, and an informal science institution, in which students investigated some of the principles of weather and physics that might help them in their sailing. In the episode presented here, students played with model boats on a small, manufactured pond. These boats function precisely the way the larger sailboats that the youth were learning to sail do, except that they do not have rudders; the only way to steer the boat is by adjusting the sails. Thus, designers conjectured that this would be a fun, slightly simplified context for exploring “points of sail”—the ideal position for a sail in coordination with orientation toward the wind.

Initial curricular design included a discussion of the proper points of sail before introducing the model boats, but one instructor decided that it might be more meaningful for the students to derive the points of sail themselves from observing the model boats. In essence, this on-the-fly adjustment was aimed at addressing a common observation from classroom studies that the “use of direct instruction in the face of a novel toy or problem can limit exploration and learning” (Weisberg et al., 2013). This is in contrast with informal environments such as informal science institutions or makerspaces where “we see amazing focus, creativity, persistence, and pride developing in people of all ages as they draw on their understanding and imagination to develop and pursue an idea and to make something concrete (even if ephemeral) that represents their ideas and understanding” (Petrich et al., 2013, 51-52).

Theoretical framework

Where cognitive learning theories such as Constructivism view learning as “building knowledge structures irrespective of the circumstances of the learning” (Papert & Harel, 1991), situative theorists consider learning to be inextricably tied to the circumstances of learning, which include the learner’s prior experiences as well as the tools and other resources of the learning environment (Werscht, 1998). From this perspective, the tools and materials, but also the goals and participation structures, fundamentally change the nature of what is learned. I take a situative perspective in this analysis, with particular attention to another important affordance of so-called “authentic” learning environments—not just the what, but also the why:

“For instance, whereas a hands-on circuit activity might be employed in a classroom to teach about electricity, the circuitry knowledge we observed in the makerspaces was used to make a night-light, customize a bike, fix a game controller, and photograph the Earth from space” (Sheridan et al., 2014, 528).

In order to provide an account of the depth of the phenomenon, I use principles of Interaction Analysis (Jordan & Henderson, 1995) to develop small-scale theory largely from video data.

Data collection and analysis

Camp involved twenty youth who moved between whole-group engagement and frequent partner work. In particular, group discussion was encouraged during the “science” portion of the curriculum, and the sailboats they were learning to operate virtually required two sailors. The model boat episode was introduced as a collaborative group activity, but the students quickly paired themselves off. The activity was also introduced by the instructors as an opportunity to try to “figure out” how the sail position influences the direction that the boat sails in, and challenged students to try to sail their boats all the way across the pond. Almost none of the students appeared to engage in either of these activities at first, however.

Data collection was designed as pilot data for a broader study. As such, all sampling was based on convenience. From within the four focal students represented in the corpus, the present case was selected because the student wearing the camera consistently narrated his own thinking to his partner, almost as he might have during a think-aloud interview protocol. Review of other focal cases suggests that this student was not anomalous in general engagement, only more forthcoming in his narration. Still, future research will aim to confirm generalizability by comparing more cases. The focal student in this case—Wally (a pseudonym)—chose to work with one other student during the model boat activity. Analysis is confined to Wally’s contributions, however, since I had not obtained consent for analyzing the other student’s participation.

Case study

Wally initially appeared to ignore instructors’ suggestions to try to sail the model boat directly across the pond, or to make any systematic investigation into how the boat moves. Over the course of fifteen minutes, he watched the boat take off from the side of the pond, ran around to retrieve it, and watched it sail off again, without making any substantial adjustments to sail position. They did adjust the direction the boat was pointed in when it was released—it appeared that the students thought this might influence the movement of the boat—but in general they were relatively content to try the same thing again and again. As the activity progressed, Wally began commenting on this confusion, albeit in a friendly way, saying things like “our boat is magic—it just turns to the side all the time”. This sense that the reasons for the behavior of the boat are an impenetrable “black box”—that the boat itself is “magic”—would be a troublesome place to leave the lesson from the perspective of the stated learning goals. At roughly fifteen minutes into their exploration, however, there was some commotion among the students. It seemed one of the students had declared himself a pirate and started trying to attack other boats. At this point, Wally began to more concertedly focus on adjusting the sail position and make observations about the resulting movement of the boat. It seemed that the introduction of piracy—and the resulting goals to either counter-attack or retreat—provided an emergent urgency, after which Wally attempted a more systematic exploration of the affordances of the model boats.

Findings and next steps

Comparison of this case with the other three focal cases revealed a similar pattern—students began the activity by simply “messing around” (Ito et al, 2009) with the model boats, ignoring the adults’ framing of the activity for the most part. It was only after the emergent social goal of dealing with the pirate became salient that students made a more concerted effort to systematically test and catalog small changes in their efforts to sail the model boats.

This finding is especially important for two reasons. First, object play has been studied extensively in young children, though it is frequently assumed to drop off, and its educational utility is less clear for late elementary and middle school students. This episode suggests that the typical progression—from “what can this object do?” to “what can I do with it?” (Hutt, 1966)—holds for older youth as well, at least when emergent goals are present. Furthermore, instructors present during this model boat activity quickly called the episode to a close once the piracy appeared, since it seemed to them to be getting out of hand (and they did not want the model boats being damaged by intentional crashes). It was only upon revisiting the data that it became clear that this emergent social structure was actually facilitating deeper engagement with precisely the kind of observations the activity was meant to target in the first place.

Paper three: It's a pizza!: Invented problems and meanings in early childhood mathematical play

Paul N. Reimer

Overview and background

Enactive and embodied perspectives shed new light on the importance of early childhood play. Vygotsky (1978) suggested play was derived from imaginative situations wherein rules emerge as children negotiate their actions based on situational constraints. Recent views of cognition imply a similar reliance on objects, materials, movements, and interactions to assist learners in forming meaningful interpretations of their activity (Núñez, Edwards, & Matos, 1999; Varela, Thompson, & Rosch, 1991). These perspectives are uniquely suited to provide explanations for meaning-making in early childhood play, as they have highlighted the interconnectedness of these resources and, drawing on biological roots, have suggested that human bodily experiences form the basis of conceptual understanding.

Early childhood educators face challenges when attempting to enact play-based pedagogies. Head Start programs, in particular, encounter a tension when considering opportunities for children's self-directed play in light of accountability for children's academic readiness (Walter & Lippard, 2017). To address this challenge, we are engaged in a multiyear partnership with two Head Start preschool centers to explore opportunities for informal mathematics learning in various seeded interest areas that encourage mathematical play (Wager, 2013). These interest areas provide a rich context to study 1) how young children interact with simple, yet carefully-designed objects, 2) the meanings children develop as they interact with materials and peers in play, and 3) the various roles adults enact as they seek to participate in play spaces with children. In particular, we are interested in how children decide what to do with objects and how they use novel ways to generate norms in their play. This work is held together by a persistent tension between what exists and what is possible: "The combination of a concrete embodied situation with alienated virtual meaning is the freedom-engendering paradox of play" (Stewart, Gapenne, & Di Paolo, 2010, p. 77).

Data collection and analysis

For this study, we analyzed preschool children's (n=12) play as they engaged in sessions of self-directed play in several interest areas within preschool classrooms at two Head Start centers. In particular, we were interested in how children attended to existing constructions or comments made by adults or peers, how children used these to develop norms of play, how they embraced ambiguity by giving new meanings to existing creations, and how their actions arose from the designed objects or ideas they developed while interacting with the objects.

Case study

In one play session, two adults invited children to play. One adult made a star with three red hexagons and silently moved it into the playspace. The adults played with the blocks, typically creating small designs out of adjacently placed blocks of different colors and shapes. Children began to play with the blocks by collecting blocks of the same color. One child decided blocks needed to touch on congruent sides and began to place blocks side by side. Another picked up blocks and enacted movements with them appropriate for the meaning he had attached to the objects. For, example, he said “I'm making a PacMan” as he held up a block and moved it in a chomping manner. After a few moments, he began to negotiate a balancing activity by stacking blocks on top of each other.

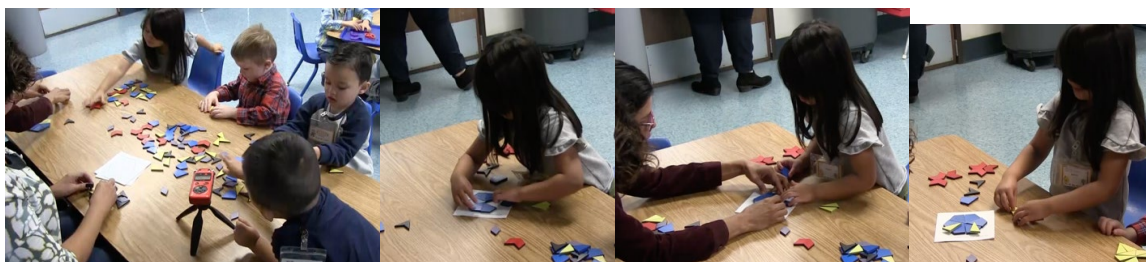


Figure 1. From left: Dalia joining play and taking the star, but in moving it the star comes apart; Dalia orienting blocks within a boundary; Dalia coordinating actions with adult; Dalia adding yellow whiskers to make a “kitty-cat”.

After two minutes of play, Dalia and another child joined the playspace (Figure 1). Dalia noticed the star the adult had made. Dalia took the star, but in moving it the star came apart. Through several attempts she put it back together. Dalia then created another copy of the star, rotating pieces to determine how to form the star. She then used a similar approach with the yellow blocks, rotating the blocks to tile. The adult asked her, “What are you making?” Dalia answered, “A pizza.” Dalia then completed a tiling with blue blocks and put two yellow triangles on top of the completed blue block figure. “It’s gonna be a kitty-cat,” she said.

Findings and next steps

Analysis of this play interaction suggests that Dalia’s play was led by both meaning and actions derived from the objects (e.g., Do these fit together?) and imagined meanings (e.g., “It’s a pizza.”). Our results suggest that an adult may enact roles in children’s play that support these developing meanings 1) as a co-enactor in play, 2) as a reflective partner who provides feedback-oriented prompts (e.g., Does that fit?), 3) as a co-participant in action through coordinated movement (e.g., A “many hands” approach), or 4) as a re-opener of play (e.g., What might happen if?). We concluded that opportunities for mathematical thinking existed in carefully-designed interest areas that allowed children to enact playful interpretations of activity based on bodily interactions with objects and imagined meanings.

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